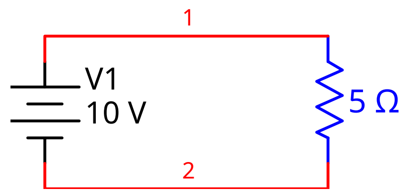
1.-OHM'S LAW: VOLTAGE, CURRENT AND RESISTANCE

Ohm's law expresses the relationship between voltage (V), current (I) and resistance (R) in a DC electrical circuit. Establishing the formula V=R*I. These relationships establish that:

If V is increased, I will increase. If V is decreased, I will decrease. If R is increased, I will decrease. If R is decreased, I will increase.

Exercise 1.1

According to the circuit, how much current would an applied voltage of 10 volts across a 5 ohm resistor produce?



Solution:

Step 1: Since the unknown is the current, we clear I

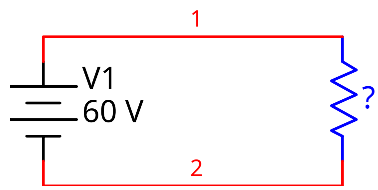
$$Yo = \frac{V}{R}$$

Step 2: We substitute the known values into the equation and obtain I.

$$Yo = \frac{V}{R} = \frac{10 volts}{5 ohms} = 2 amperes$$

Exercise 1.2

According to the diagram, what is the resistance that, if a voltage of 60 volts is applied to it, would produce a current of 3 amperes?



Solution

Step 1: Since the unknown is the resistance, we clear R

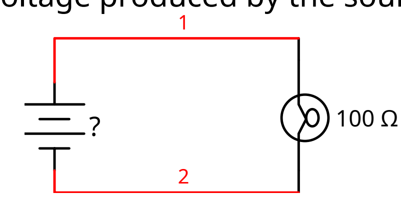
$$R = \frac{V}{Yo}$$

Step 2: We substitute the known values into the equation and obtain R.

$R = \frac{V}{Yo} = \frac{60 volts}{6 amperes} = 10 ohms$

Exercise 1.3

If the source in the circuit diagram has a resistance of 100 ohms and a current of 1 ampere, what will be the voltage produced by the source?



Solution:

Step 1: Since the unknown in this case is the voltage, we clear V. $V = RI$

Step 2: We substitute the known values into the equation and obtain I. $V = RI = 100 ohms * 1 ampere = 100 Volts$

2.-SERIES CIRCUITS

2.1 SERIES RESISTORS

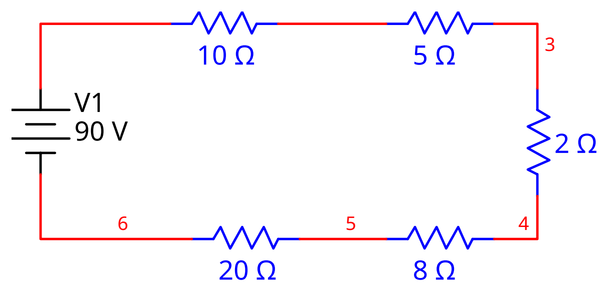
A series circuit is made up of a set of loads or resistors through which the total current from the source flows in a single path and there are no divisions between these loads, so the current is the same at any point.

To calculate either the current or voltage in a circuit with series loads, all the loads or resistances are first added together to form a total or equivalent resistance and from there the other variables are calculated using Ohm's law. Therefore the total resistance of a series circuit is calculated as follows:

Rtotal=R1+R2+R3+R4+R5+...Rn

Exercise 2.1.1

Calculate the total current that flows in the following circuit with series loads, considering that the source is 90 volts.



Solution:

Step 1: First we add all the resistances to obtain the equivalent $Total = 10\Omega + 5\Omega + 2\Omega + 8\Omega + 20\Omega$

$Total = 45\Omega$

Step 2: Now since the unknown is the current, we clear I from Ohm's law equation and substitute.

$$Yo = \frac{V}{R}$$

$$Yo = \frac{90\,V}{Total} = \frac{90\,V}{45\Omega} = 2\,amperes$$

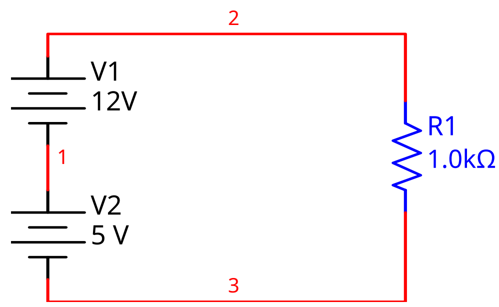
2.2 DC SERIES SOURCES

Voltage sources can also be placed in series, so the total voltage in a circuit where there are two or more sources in series is the sum of the individual voltages of each source.

When the polarities of the sources are in the same direction, their voltage adds, when their polarities are in opposite directions, they subtract.

Exercise 2.2.1

For the following circuit, calculate the current supplied by the two sources in series.



Solution:

Step 1: First we need to get the total voltage of the circuit, so we need to add or subtract the voltage sources. From the arrangement of the DC sources we can deduce that they are adding since their polarities point in the same direction (the positive part points up, and the negative part points down). Another way to know is by observing the part where the two sources are joined; if they have different polarities at the junction, they are added, if they are equal polarities, they are subtracted.

Therefore, the following are added:

$$Vtotal = 12V + 5V$$

$$Vtotal = 17V$$

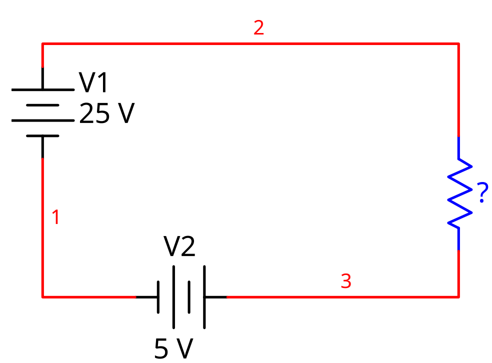
Step 2: Once the total voltage is obtained, we can clear I from Ohm's law equation and obtain the total current supplied by the two sources.

$$Yo = \frac{V}{R} = \frac{Vtotal}{1k\Omega}$$

$$Yo = \frac{17V}{1k\Omega} = 17mA$$

Exercise 2.2.2

Obtain the value of the circuit resistance so that a current of 2.5A flows if there are two sources in series with their respective values, as shown in the diagram:



Solution:

Step 1: Obtain the total voltage. We can observe that at the point where the two sources are joined they have the same polarity, that is, the negative of the source

One is connected to the negative of source two. Therefore they are subtracted. It is always best to subtract the lower voltage from the higher voltage source.

In this way the total voltage is as follows:

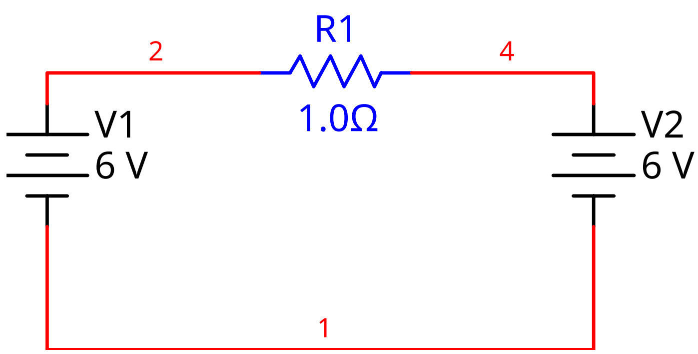
$$Vtotal = 25V - 5V$$
$$Vtotal = 20V$$

Step 2: Calculate the resistance from Ohm's law with the known data.

$$R = \frac{V}{Yo} = \frac{Vtotal}{2.5TO}$$
$$R = \frac{20V}{2.5TO} = 8\Omega$$

Exercise 2.2.3

Calculate the current flowing through a series circuit that has a load resistance of 1 ohm and two direct voltage sources arranged as shown in the circuit shown:



Step 1: First, calculate the total voltage of the circuit. To do this, we observe the arrangement of the sources. It can be seen that both are of the same value, however, the points where they are joined are of the same pole, therefore they are being subtracted. Consequently, when subtracting them, we will have 0V and therefore there will be no current circulation.

$$Vtotal = 6V - 6V = 0V$$
$$Yo = \frac{V}{R} = \frac{0V}{1\Omega} = 0TO$$

3.-PARALLEL CIRCUITS

A parallel circuit is one in which there are one or more points where the current divides and follows different paths.

3.1 PARALLEL RESISTORS

For parallel resistors, three main rules can be observed to calculate the equivalent resistance:

- For a given number of resistors in parallel and of the SAME VALUE, the total resistance is calculated by dividing the value of a single resistor by the number of them.

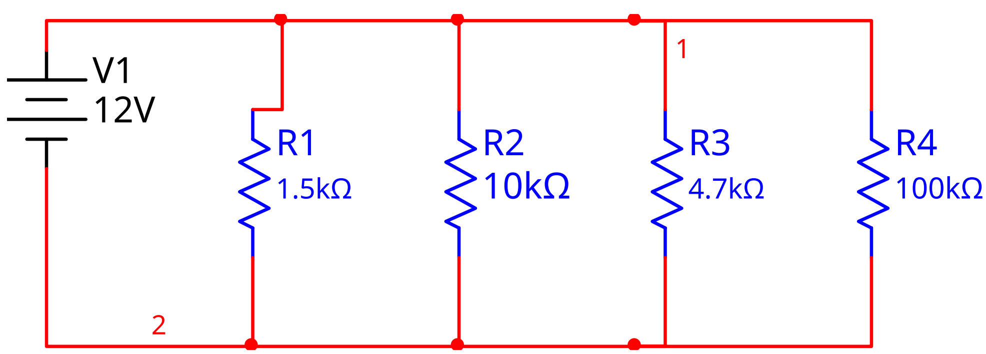
- The total resistance of two resistors in parallel of equal or different values can be calculated with the formula:**Rt = (R1*R2) / R1+R2**

- To calculate the equivalent resistance of any number of resistors with different or equal values, the following formula is used:

$$Total = \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4} + \ldots + \frac{1}{Rn}}$$

Exercise 3.1.1

Find the current flowing through the circuit shown, assuming a 12V source.



Solution:

This example can be solved in two ways, by calculating the current that flows through each resistor and adding them together, or by calculating the equivalent resistance and obtaining the total current. We will proceed to solve it by both methods to demonstrate that the same results are obtained.

Method 1: Calculating individual currents

Step 1: In a parallel circuit the voltage remains constant between each division or branch, so from the voltage and resistance the current that flows through each branch can be calculated using Ohm's law.

$$Yo_1 = \frac{V}{R1} = \frac{12V}{1.5k\Omega} = 8mA$$

$$Yo_2 = \frac{V}{R2} = \frac{12V}{10k\Omega} = 1.2mA$$

$$Yo_3 = \frac{V}{R3} = \frac{12V}{4.7k\Omega} = 2.55mA$$

$$Yo_4 = \frac{V}{R4} = \frac{12V}{100k\Omega} = 0.12mA$$

Step 2: Since the total current is the sum of the individual currents, we obtain the current that circulates in the circuit:

$$Total = Yo_1 + Yo_2 + Yo_3 + Yo_4$$

$$Total = 8mA + 1.2mA + 2.55mA + 0.12mA$$

$$Itotal = 11.87mA$$

Method 2: Calculating the total resistance

Step 1: Using the sum of reciprocals we calculate the total resistance.

$$Total = \frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4}}$$

$$Total = \frac{1}{\frac{1}{1.5k\Omega} + \frac{1}{10k\Omega} + \frac{1}{4.7k\Omega} + \frac{1}{100k\Omega}} = 1.01k\Omega$$

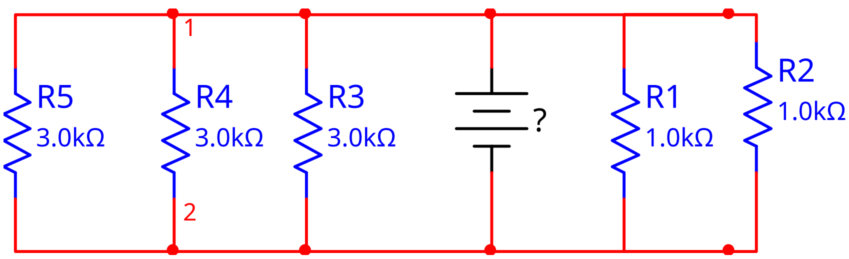
Step 2: Now using Ohm's law we calculate the total current.

$$Yo = \frac{V}{Total} = \frac{12V}{1.01k\Omega} = 11.88mA$$

As we can see from both methods we arrive at the same result.

Exercise 3.1.2

Calculate the voltage provided by the source so that there is a current of 6 amperes flowing through the entire circuit according to the diagram.



Solution:

Step 1: Calculate the equivalent resistance. We observe that each pair of resistors has the same value. Therefore, we can apply the product/sum formula to calculate the equivalent resistance of each pair or the formula for resistors of the same value.

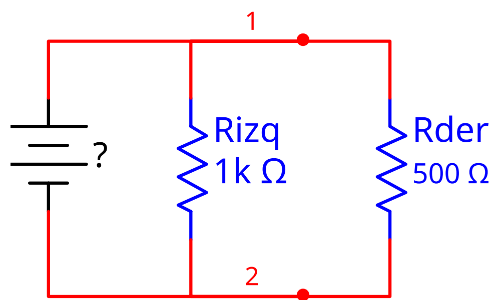
Step 2: Calculate the torque on the right side of the source:

$$Rder = \frac{R1\,R2}{R1+R2} = \frac{1k*1k}{1k+1k} = 500\Omega$$

Step 3: Calculate the torque on the left side of the source:

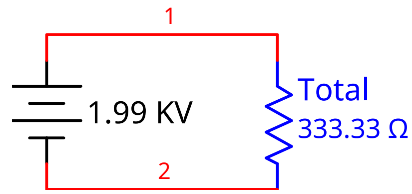
$$Rizq = \frac{3k}{3} = 1k\Omega$$

Step 4: Once we have the circuit reduced to two resistors as seen in the diagram, we calculate the equivalent resistance:



$$Total = \frac{RderRiaq}{Rder+Rizq} = \frac{1k*500}{1k+500} = 333.3\Omega$$

Step 5: Once the total resistance has been calculated, we proceed to obtain the source voltage using Ohm's law:



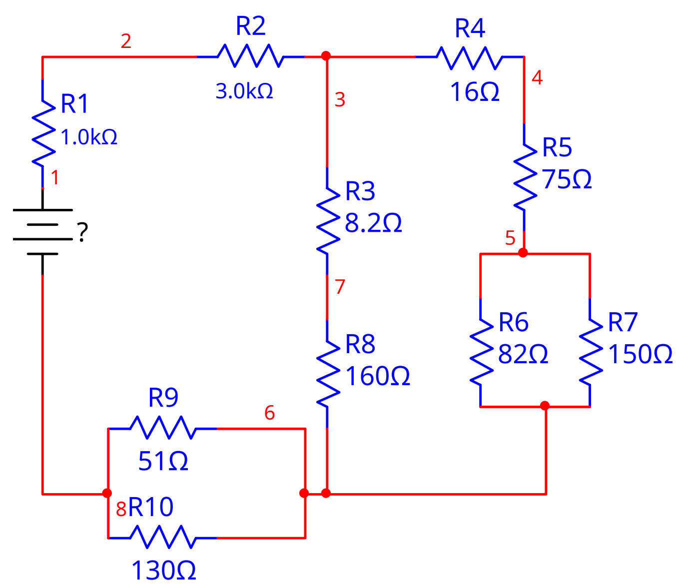
$V = R * Yo = 333.33\Omega * 6TO = 1.99 KVolts$

4.-MIXED CIRCUITS: SERIES AND PARALLEL

A mixed circuit is one that contains elements both in series and in parallel, through which a current flows.

Exercise 4.1

Determine the voltage provided by the source in the following circuit, if there is a circulating current of 60 mA:



Solution:

Step 1: We start by reducing from the part furthest from the source, firstly by the parallels, in this case we start with R6 and R7

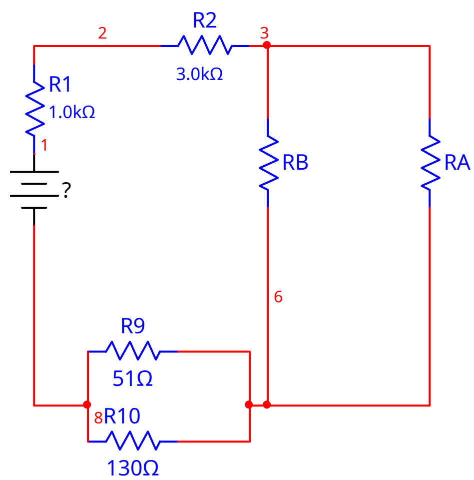
$$R67 = \frac{R6 * R7}{R6 + R7} = \frac{82 * 150}{82 + 150} = 53.01\Omega$$

Step 2: Now that the equivalent resistance of R6 and R7 is in series, it is added to the series resistances R4 and R5.

$$RA = R4 + R5 + R67 = 16 + 75 + 53.01 = 144.01\Omega$$

Step 3: We then add the series resistors R3 and R8 to later add them in parallel with RA.

$$RB = R3 + R8 = 8.2 + 160 = 168.2\Omega$$



Step 4: Now we make the parallel between the resistors RA and RB:

$$RAB = \frac{RA * RB}{RA + RB} = \frac{144.01*168.2}{144.01+168.2} = 77.58\Omega$$

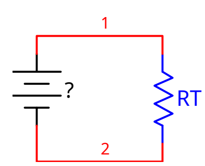
Step 5: We carry out the parallel of R9 and R10:

$$RC = \frac{R9 * R10}{R9 + R10} = \frac{51*130}{51+130} = 36.62\Omega$$

Step 6: Now that all the resistors are in series, we are ready to add them together to obtain the total equivalent resistance:

$$RT = R1 + R2 + RAB + RC$$

$$RT = 1K + 3K + 77.58 + 36.62 = 4114.2\Omega$$



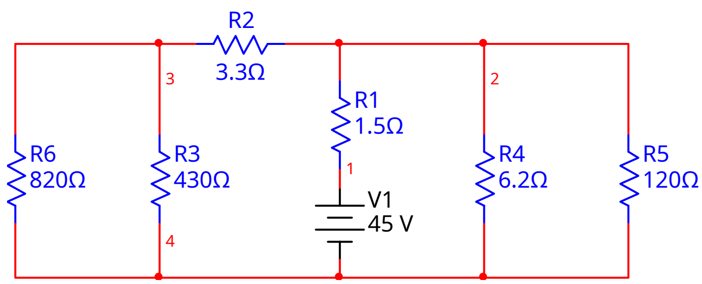
Step 7: Finally we calculate the source voltage using Ohm's law.

$V = RI$

$V = 4114.2 * 60\,mA = 246.85\,V$

Exercise 4.2

Find the current supplied by the 45V source in the circuit shown:



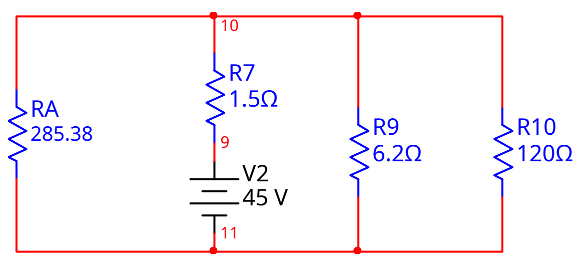
Solution:

Step 1: We solve the parallel of R6 and R3

$$R63 = \frac{R6 * R3}{R6 + R3} = \frac{820 * 430}{820 + 430} = 282.08\Omega$$

Step 2: We add the previous parallel in series with R2

$$RA = R63 + R2 = 282.08 + 3.3 = 285.38\Omega$$

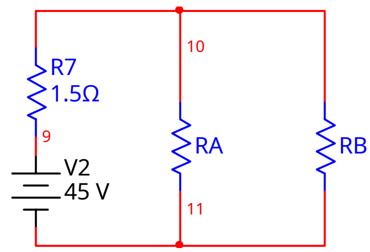


Step 3: We solve the parallel of R9 and R10

$$RB = \frac{R9 * R10}{R9 + R10} = \frac{6.2 * 120}{6.2 + 120} = 5.89\Omega$$

Step 4: Now we see that RA and RB are in parallel so we add them that way.

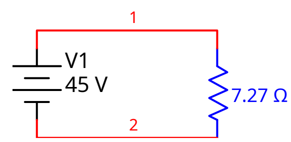
$$RAB = \frac{RA * RB}{RA + RB} = \frac{285.38 * 5.89}{285.38 + 5.89} = 5.77\Omega$$



Step 5: Now that we have the two resistors in series, we are ready to add them together and obtain the total equivalent resistance:

$$RT = R7 + RAB = 1.5 + 5.77 = 7.27\Omega$$

Step 6: We proceed to obtain the current using Ohm's law.



$$Yo = \frac{V}{R} = \frac{45V}{7.27\Omega} = 6.18 TO$$

5.- POWER

5.1 POWER IN SERIES AND PARALLEL CIRCUITS

The power of an element is expressed as the result of multiplying the current that flows through it and the voltage applied at its ends, obtaining the following formula:

P=V*I

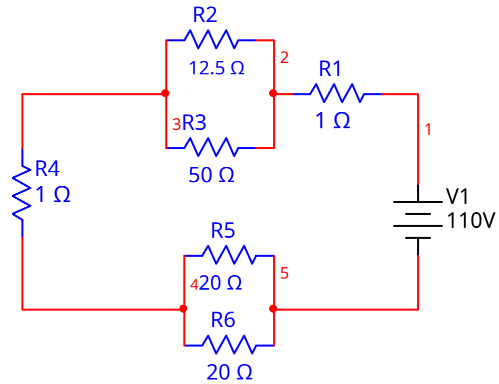
The way resistance relates to power is expressed in the following formulas:

$$P = R * Yo^2$$

$$P = \frac{V^2}{R}$$

Exercise 5.1.1

We have the following mixed circuit, which is powered by a 110V DC source. Calculate the current, voltage and individual power for each resistor.



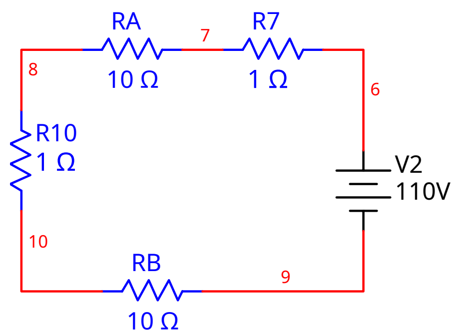
Solution:

Step 1: We start by finding the total current, so we calculate the equivalent resistance of the entire circuit:

We solve the parallels:

$$Ra = \frac{R2 * R3}{R2 + R3} = \frac{12.5 * 50}{12.5 + 50} = 10\Omega$$

$$Rb = \frac{20}{2} = 10\Omega$$

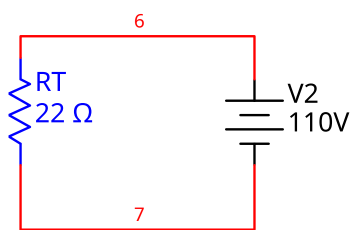


We add the resistors in series:

$$RT = Ra + Rb + R1 + R4 = 10 + 10 + 1 + 1 = 22\Omega$$

And we calculate the total current:

$$Yo = \frac{V}{RT} = \frac{110V}{22\Omega} = 5TO$$



Step 2: We calculate the individual voltages and powers for the resistors that are originally in series, (R1 and R4), since the current in these is the same:

For R1:

$V = R * Yo = 1\Omega * 5TO = 5V P = V *$

$Yo = 5V * 5TO = 25 Watts$

For R4, as its value is the same as R1 and being in series it has the same current value, therefore its voltage is 5V and its power is 25 Watts.

Step 3: Now we move on to parallel resistors, starting with R2 and R3. As we know, in a parallel arrangement the current is divided, but the voltage remains the same, so from its series equivalent of 10 ohms we can obtain the voltage as follows:

$$V = R * Yo = 10\Omega * 5\,TO = 50\,V$$

Step 4: From the common voltage for each resistor, we calculate its individual current and hence its power.

For R2:

$$Yo = \frac{V}{R2} = \frac{50}{12.5} = 4\,TO$$
$$P = V * Yo = 50 * 4 = 200\,w$$

For R3:

$$Yo = \frac{V}{R3} = \frac{50}{50} = 1\,TO$$
$$P = V * Yo = 50 * 1 = 50\,w$$

Step 5: We repeat the same procedure for the parallel of R5 and R6.

We calculate its voltage from its series equivalent:

$$V = R * Yo = 10\Omega * 5\,TO = 50\,V$$

Now we calculate current and power for each resistor in parallel:

For R5:

$$Yo = \frac{V}{R5} = \frac{50}{20} = 2.5\,TO$$
$$P = V * Yo = 50 * 2.5 = 125\,w$$

For R6:

$$Yo = \frac{V}{R6} = \frac{50}{20} = 2.5\,TO$$
$$P = V * Yo = 50 * 2.5 = 125\,w$$

Step 6: We make a table with all the individual values, and we check that the sum of all the individual powers is equal to the total power.

Ptotal=V*I=110V*5A=550W

R1=1 ohm	V1=5V	I1=5A	P1=25 w
R2=12.5 ohms	V2=50V	I2=4A	P2=200 w
R3=50 ohms	V3=50V	I3=1A	P3=50w
R4=1 ohm	V4=5V	I4=5A	P4=25 w
R5= 20 ohms	V5=50V	I5=2.5A	P5=125 w
R6=20 ohms	V6=50V	I6=2.5A	P6=125 w
			Total=550W

5.2 MAXIMUM POWER TRANSFER

Maximum power transfer refers to when, for example, an active network, in this case a battery, has an internal resistance of a certain value and for it to transfer its maximum power or efficiency to a load, this has to be the same value as the internal resistance.

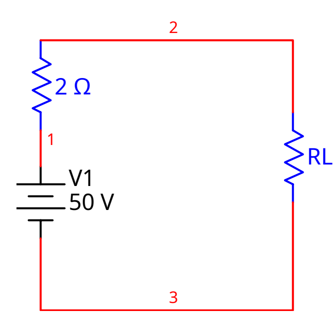
The power transmitted to the load can be calculated in two ways:

1) Calculating the current first: $Yo = \frac{V}{R+RL}$ and then calculate the power individual in resistance: $P = Yo^2 RL$
2) Using the following formula directly:

$$P = \frac{V^2 RL}{(Ri+RL)^2}$$

Exercise 5.2.1

A 50V battery has an internal resistance (Ri) of 2 ohms. Demonstrate the maximum power transfer theorem by calculating the power transmitted to the load (RL) when it is 0.5 ohm, 2 ohms, and 6 ohms.



Solution:

Step 1: We calculate the current for a RL=0.5 ohm using the first method:

$$Yo = \frac{V}{Ri + RL} = \frac{50\,V}{2\Omega + 0.5\Omega} = \frac{50\,V}{2.5\Omega} = 20\,TO$$

Step 2: Now we calculate the individual power of RL:

$$P = Yo^2 RL = (20\,TO)^2 (0.5\Omega) = 200\,W$$

Step 3: We then calculate the power for a 2 ohm load, using the direct formula:

$$P = \frac{V^2 RL}{(Ri + RL)^2} = \frac{(50\,V)^2 (2\Omega)}{(2\Omega + 2\Omega)^2} = 312.5\,W$$

Step 4: Now we do the calculation for a 6 ohm load resistance with direct formula:

$$P = \frac{V^2 RL}{(Ri + RL)^2} = \frac{(50\,V)^2 (6\Omega)}{(2\Omega + 6\Omega)^2} = 234.375\,W$$

Step 5: We analyze the results with the three load values and see that the maximum power in the load is obtained when Ri=RL, in this case when it is 2 ohms. When the load resistance is higher or lower, the power obtained will always be lower than that obtained with the 2 ohms resistance, which was 312.5 Watts.

RL=0.5 ohms	P=200W
RL=2 ohms	P=312.5W
RL=6 ohms	P=234.375W